

# Домашна работа

№39

$$v) \operatorname{div}(\vec{A} \times \vec{B}) = \vec{B} \operatorname{rot} \vec{A} - \vec{A} \operatorname{rot} \vec{B}$$

$$\begin{aligned} 1) \operatorname{div}(\vec{A} \times \vec{B}) &= \nabla \cdot (\vec{A} \times \vec{B}) + \nabla \cdot (\vec{A} \cdot \vec{B}) = \\ &= \vec{B} \cdot (\nabla \times \vec{A}) + \vec{A} \cdot (\nabla \times \vec{B}) = \vec{B} \operatorname{rot} \vec{A} + \vec{A} (\vec{B} \times \nabla) = \\ &= \vec{B} \operatorname{rot} \vec{A} - \vec{A} \operatorname{rot} \vec{B} \end{aligned}$$

$$\begin{aligned} g) \operatorname{rot}(\vec{A} \times \vec{B}) &= \vec{A} \operatorname{div} \vec{B} - \vec{B} \operatorname{div} \vec{A} + (\vec{B} \cdot \nabla) \vec{A} - (\vec{A} \cdot \nabla) \vec{B} \\ \operatorname{rot}(\vec{A} \times \vec{B}) &= [\nabla \times (\vec{A} \times \vec{B})] = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) \\ &= (\vec{A} \cdot \nabla) \vec{B} - \vec{B} \operatorname{div} \vec{A} + \vec{A} \operatorname{div} \vec{B} - (\vec{B} \cdot \nabla) \vec{A} \end{aligned}$$

$$\begin{aligned} e) \operatorname{grad}(\vec{A} \cdot \vec{B}) &= \vec{A} \times \operatorname{rot} \vec{B} + \vec{B} \times \operatorname{rot} \vec{A} + (\vec{B} \cdot \nabla) \vec{A} + (\vec{A} \cdot \nabla) \vec{B} \\ \operatorname{grad}(\vec{A} \cdot \vec{B}) &= \nabla(\vec{A} \cdot \vec{B}) + \nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + (\vec{B} \cdot \nabla) \vec{A} + \\ &+ \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla) \vec{B} = \vec{A} \times \operatorname{rot} \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{B} \times \operatorname{rot} \vec{A} + (\vec{A} \cdot \nabla) \vec{B} \end{aligned}$$

№40

$$e) (\nabla \times \vec{A}) \times \vec{B} = \vec{A} \operatorname{div} \vec{B} - (\vec{A} \cdot \nabla) \vec{B} - \vec{A} \times \operatorname{rot} \vec{B} - \vec{B} \times \operatorname{rot} \vec{A}$$

$$\begin{aligned} (\nabla \times \vec{A}) \times \vec{B} &= (\nabla \times \vec{A}) \times \vec{B} + (\nabla \times \vec{A}) \times \vec{B} = (\nabla \times \vec{A}) \times \vec{B} - (\vec{A} \cdot \nabla) \vec{B} \\ &= -\vec{B} \times (\nabla \times \vec{A}) - (\vec{A} \cdot \nabla) \vec{B} - \vec{A} \times (\nabla \times \vec{B}) + \vec{A} \cdot (\nabla \cdot \vec{B}) = \\ &= \vec{A} \operatorname{div} \vec{B} - (\vec{A} \cdot \nabla) \vec{B} - \vec{A} \times \operatorname{rot} \vec{B} - \vec{B} \times \operatorname{rot} \vec{A} \end{aligned}$$